$R + R^2$ supergravity and off-shell multiplets

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mostly based on collaborations
with **R. Kallosh and S. Ferrara**

Dubna workshop
'What next ?'
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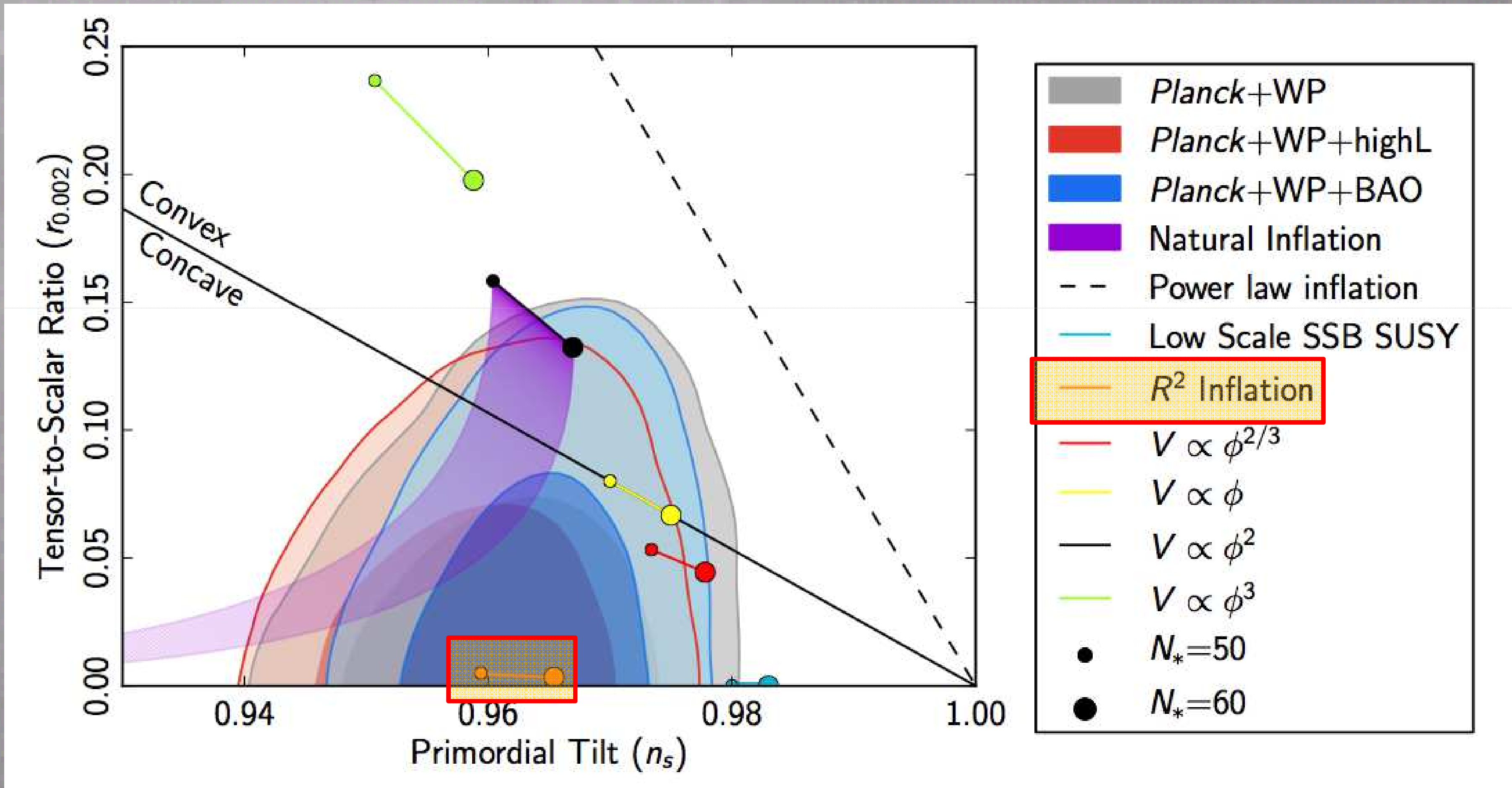
Planck 2013 results; constraints on inflation



Planck collaboration, 1303.5082

Starobinsky inflation and higher derivative terms in gravity

- Higher derivative terms to replace initial singularity (Starobinsky, 1980);
  rewritten as R+R^2 : Kofman, Linde, Starobinsky, 1985
- Correcting Einstein theory (2nd order Lagrangian) with 4th order terms

$$S=\int \mathrm{d}^4x\,\sqrt{g}L\,,\qquad L=\frac{1}{2\kappa^2}R$$

$$\kappa^2=8\pi G$$

$$L=\frac{1}{2\kappa^2}R+a\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+b\,R_{\mu\nu}R^{\mu\nu}+c\,R^2$$

- These are also first string corrections
  (and also studied for black hole entropy, corrections to AdS/CFT, quantum loops, ...)

Plan

1. Gravity: 4th order derivative terms and conformal structure
2. N=1 supergravity: auxiliary fields and supersymmetrization of $R+R^2$ terms
3. N=2 auxiliary fields and expectations for dual theories to $R+R^2$ supergravity
4. N=4 superconformal structure
5. Conclusions

4th order terms in gravity

- Euler density $L = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$
  is a topological invariant
  $\rightarrow$ does not contribute to field equations.
- Conformal invariant
$$\begin{aligned} L &= C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \\ &= R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \tfrac{1}{3}R^2 \\ &\simeq 2(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2) \end{aligned}$$
- We parametrize
$$\begin{aligned} L &= \frac{1}{2\kappa^2}R + a\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + b\,R_{\mu\nu}R^{\mu\nu} + c\,R^2 \\ &= \frac{1}{2\kappa^2}R + \alpha R^2 + \beta(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2) \end{aligned}$$
- K. Stelle, 1977:
  - without α and β: massless spin 2
  - with α: additional physical spin 0 with $\boldsymbol{m_0^{-2} = 12\,\alpha\,\kappa^2}$
  - with β : additional ghost spin 2 with $\boldsymbol{m_2^{-2} = -2\beta\,\kappa^2}$

Degrees of freedom (dof)

- On-shell dof = # helicity states (4 dim)
  - massless: 2 for every spin except s=0: 1 state
  - massive : $2s+1$
- Off-shell dof = # field components − # gauge transform
  - real scalar: 1
  - Majorana fermion: 4
  - Gauge vector: 4 − 1 = 3
  - metric field : 16 − 6 − 4 = 6   (local Lorentz and translations)
  - gravitino : 16 – 4 = 12 (local susy)
- Supersymmetry: as well on-shell as off-shell
  # bosonic dof = # fermionic dof
  (only known for N=1,2):
  necessity of auxiliary fields (0 dof on shell)

Particle states

$$L = \frac{1}{2\kappa^2}R + \alpha R^2 + \beta(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2)$$

- $\frac{1}{2\kappa^2}R$ → massless spin 2
- αR^2 → massive spin 0
- $\beta(\ldots)$ → conformal → ghost massive spin 2

- A massless spin 2 has 2 degrees of freedom (dof)
- Off-shell metric field has 6 dof
- Conformal metric field (Weyl field) has 5 dof
- A massive spin 2 has 5 dof
- Poincaré action from Weyl + compensating scalar :

In higher derivative action
- Weyl → ghost
- compensating scalar → physical scalar

Goal of this talk

- Show how this structure is valid with supersymmetry
- auxiliary fields (off shell dof) become propagating
  $\Rightarrow$ structure of auxiliary fields already determines content of massive sector
  $\Rightarrow$ different auxiliary fields $\rightarrow$ different massive sector
- In conformal structure: different compensators
  - in N=1 : understanding of two inequivalent generalizations
  - in N=2: there are 3 different generalizations with different compensating multiplets, and we thus expect different massive sectors for higher-derivative theories

Conformal structure: Weyl multiplet

P_a	4	translations	e_μ^a
M_{ab}	6	Lorentz rotations	~~$\omega_\mu{}^{ab}$~~
D	1	dilatation	b_μ
K_a	4	special conformal transformations	~~$f_\mu{}^a$~~

Constraints determine two gauge fields

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,b) \qquad f_\mu^a = -\frac{1}{4}R_\mu{}^a + \frac{1}{24}e_\mu^a R$$

'Weyl multiplet': $e_\mu{}^a,\ b_\mu$

dof: $e_\mu{}^a$: 16 − 4 − 6 −1= 5

b_μ : 4 – 4 = 0

$$\delta_K b_\mu = e_{\mu a}\lambda_K^a$$

gauge choice : $b_\mu = 0$

Conformal action

Consider a scalar. We have to define dilatation transformation

$$\delta_D\phi = w\lambda_D\phi$$

w parameter: 'Weyl weight'

$$\Box^C\phi \equiv \eta^{ab}\mathcal{D}_b\mathcal{D}_a\phi = \Box\phi + 2w\, e^{a\mu} f_{\mu a}\phi \qquad e^{\mu a} f_{\mu a} = -\frac{1}{12}R$$

$$w = 1 \qquad \Box^C\phi = \Box\phi - \tfrac{1}{6}R\phi \qquad \delta_{\mathrm{D,K}}\Box^C\phi = 3\lambda_D\Box^C\phi\,.$$

$$I = -\frac{1}{2}\int \mathrm{d}^4x\, e\phi\Box^C\phi = \int \mathrm{d}^4x\, e\left(\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{1}{12}R\phi^2\right)$$

↓

wrong sign kinetic terms;
but this is not physical

gauge fix dilatations $\phi^2 = \frac{6}{\kappa^2} \;:\; I = \int \mathrm{d}^4x\, e\frac{1}{2\kappa^2}R$

Higher order action

- The non-conformal action $L = \frac{1}{2\kappa^2}R + \alpha R^2$

with previous gauge $\phi = \frac{\sqrt{6}}{\kappa}, \qquad \Box^C\phi = -\frac{1}{\sqrt{6}\kappa}R\,, \qquad \frac{\Box^C\phi}{\phi} = -\frac{1}{6}R$

is the conformal action $L = -\frac{1}{2}\phi\Box^C\phi + 36\alpha\left(\frac{\Box^C\phi}{\phi}\right)^2$

- Conformal dualization: $L = -\frac{1}{2}\phi\Box^C\phi + \sigma\left(\chi - \frac{\Box^C\phi}{\phi}\right) + 36\alpha\,\chi^2$

eliminate χ : $L = -(\frac{1}{2}\phi^2 + \sigma)\frac{\Box^C\phi}{\phi} - \frac{1}{144\,\alpha}\sigma^2$

D - gauge:
$\frac{1}{2}\phi^2 + \sigma = \frac{3}{\kappa^2}$

$$\begin{aligned} L &= -\frac{3}{\kappa^2}\left(\frac{\Box^C\phi}{\phi} - \frac{1}{6}R\right) - \frac{1}{144\,\alpha}\left(\frac{3}{\kappa^2} - \frac{1}{2}\phi^2\right)^2 \\ &\simeq \frac{1}{2\kappa^2}R - \frac{3(\partial_\mu\phi)(\partial^\mu\phi)}{(\kappa\phi)^2} - \frac{1}{16\,\alpha\kappa^4}\left(1 - \frac{1}{6}\kappa^2\phi^2\right)^2 \end{aligned}$$

physical normalization

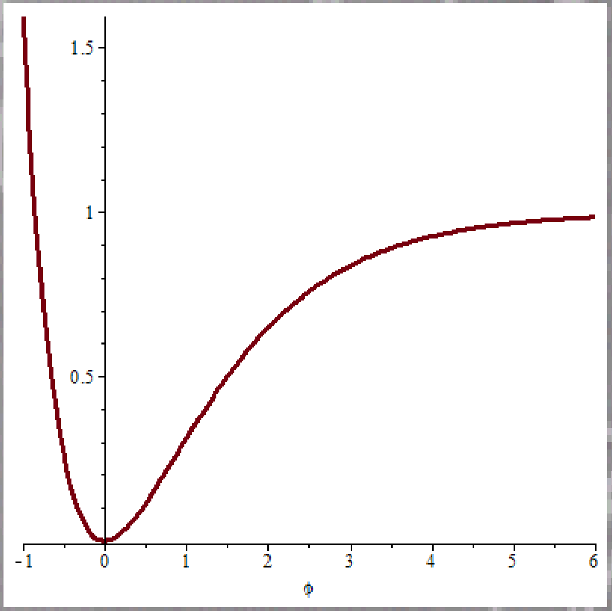
$\kappa\phi = \sqrt{6}\exp\left(-\frac{1}{\sqrt{6}}\kappa\varphi\right)$

$$L = \frac{1}{2\kappa^2}R - \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{1}{16\,\alpha\kappa^4}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\kappa\varphi\right)\right]^2$$

Starobinsky model result

$$L = \frac{1}{2\kappa^2}R - \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{1}{16\,\alpha\kappa^4}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\kappa\varphi\right)\right]^2$$

- Thus: $R + R^2$ without conformal term: equivalent to one physical propagating massive compensating field.
- Original result: B. Whitt, 1984
- Scalar with positive kinetic terms ('scalaron'), and mass as in seminal work of K. Stelle, potential suitable for slow roll inflation to Minkowski vacuum



Schematic

- off-shell Poincaré (6 dof) =

conformal spin 2 (5 dof) + compensating scalar (1 dof)

↓ Weyl action ↓ dual of R^2 action

massive spin 2 ghost + massive spin 0 physical

- on-shell Poincaré (2 dof)

R action

N=1 supergravity

- Massless multiplets:

spin	#	dof/particle	bosonic	fermionic
2	1	2	2	
3/2	1	2		2
$TOTAL$			2	2

spin	#	dof/particle	bosonic	fermionic
1	1	2	2	
1/2	1	2		2
$TOTAL$			2	2

spin	#	dof/particle	bosonic	fermionic
1/2	1	2		2
0	2	1	2	
$TOTAL$			2	2

- Massive multiplets

spin	#	dof/particle	bosonic	fermionic
2	1	5	5	
3/2	2	4		8
1	1	3	3	
$TOTAL$			8	8

spin	#	dof/particle	bosonic	fermionic
1	1	3	3	
1/2	2	2		4
0	1	1	1	
$TOTAL$			4	4

spin	#	dof/particle	bosonic	fermionic
1/2	1	2		2
0	2	1	2	
$TOTAL$			2	2

N=1 D=4 Superconformal gauge fields and the Weyl multiplet

P_a	M_{ab}	D	K_a	U(1)	Q	S
4	6	1	4	1	4	4
e_μ^a	$\omega_\mu{}^{ab}$	b_μ	$f_\mu{}^a$	A_μ	ψ_μ	ϕ_μ

determined by constraints

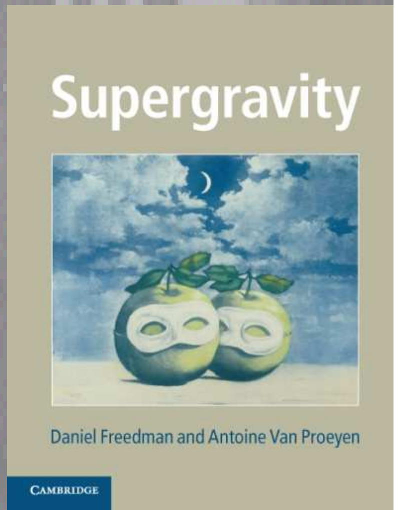
'Weyl multiplet': $e_\mu{}^a, b_\mu, A_\mu, \psi_\mu$

field	# components	spin 2	spin $\frac{3}{2}$	spin 1
$e_\mu{}^a$	$16-4-6-1=5$	1		
b_μ	$4-4=0$			
A_μ	$4-1=3$			1
ψ_μ	$16-4-4=8$		2	

is the massive mult.

spin	#	dof/particle
2	1	5
3/2	2	4
1	1	3

explicit in



Superconformal methods for cosmology, see review R. Kallosh, 1402.0527

To super-Poincaré

- super-Poincaré physical fields:

$e_\mu{}^a:\ 16-4-6=6;\quad \psi_\mu:\ 16-4=12$

- *We need auxiliary fields.*
- Other way: compare with conformal

$e_\mu{}^a:\ 16-4-6\ \underset{}{\overset{D}{-1}}=5\ ;\ \psi_\mu:\ 16-4\ \overset{S}{-\ 4}=8$

$A_\mu:\ 4\ \underset{U(1)}{-1}=5$

We need compensators: the minimal c… r 'old minimal'

- chiral mult. $(Z,\Omega,E)\longrightarrow$ aux. fields $A_\mu(4)$ en $F(2)$
- linear mult. $(L,\varphi,E_{\mu\nu})\longrightarrow$ aux. fields $A_\mu(3)$ en $E_{\mu\nu}(3)$

'new minimal'

In higher derivative theories

- If Weyl action:
  Weyl multiplet is massive spin 2 ghost multiplet.
- If R^2 terms: after dualization: compensating multiplet becomes physical massive multiplet
- old minimal (off-shell chiral mult);

field	# components	spin $\frac{1}{2}$	spin 0
X	2		2
F	2		2
Ω	4	2	

compare massive chiral mult.

spin	#
1/2	1
0	2

With old minimal we get 2 massive chiral multiplets !

Superconformal tensor calculus for old-minimal supergravity

- Poincaré supergravity: with compensating multiplet $-[S_0\bar{S}_0]_D$
- R^2 action results from kinetic multiplet $\mathcal{R}(S_0)$ that contains the curvature:

$$[-S_0\bar{S}_0 + \mathcal{R}(S_0)\overline{\mathcal{R}(S_0)}]_D$$

- Dualization

$$\left[-S_0\bar{S}_0 + S\overline{S}\right]_D + \left[S_0^2\sigma(S - \mathcal{R}(S_0))\right]_F$$
$$= \left[-S_0\bar{S}_0(1 + \sigma + \bar{\sigma}) + S\overline{S}\right]_D + \left[S_0^2\sigma S\right]_F$$

S_0: compensating: S and σ physical + potential

Conclusion on old minimal set

- Dual of $R+R^2$ supergravity is a matter coupling with 2 chiral multiplets
  (S. Cecotti; 1987; R. Kallosh, S. Ferrara, AVP, 1309.4052)
- Requires stabilization of other scalars
  (J. Ellis, D. Nanopoulos, D. Olive, 1305.1247; 1307.3537;
  R. Kallosh and A. Linde, 1306.3214)

New minimal set of auxiliary fields

- without higher derivatives: dual to old minimal:
  S. Ferrara, L. Girardello, T. Kugo and AVP, 1983

field	# components	spin 1	spin $\frac{1}{2}$	spin 0
L	1			1
E_a	3	1		
φ	4		2	

spin	#
1	1
1/2	2
0	1

- R^2 term gives different result: massive vector multiplet:
  S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, 1987;
  S. Ferrara, R. Kallosh, A. Linde and M. Porrati, 1307.7696.
- This massive multiplet was constructed in AVP, 1980.
- Only 1 scalar, does not need stabilization.
- Further possibilities for inflaton field, Kähler structure and gauged isometries in
  S. Ferrara, P. Frè, A. Sorin, 1311.5059 en 1401.1201

Conclusions

- Conformal construction of supergravity shows the content of higher derivative actions.
- Different compensating multiplets imply different massive multiplets that are dual to $R+R^2$ actions.
- N=1 : known since old papers of S. Cecotti and S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, 1987.
  Especially new-minimal interesting dual:
  a massive vector multiplet with only one physical scalar.
  Recently studied in more detail in view of cosmology application.
- N=2: different auxiliary field formulations known.
  They may lead to different physical $R+R^2$ theories.
- N=4: probably no $R+R^2$ theory.